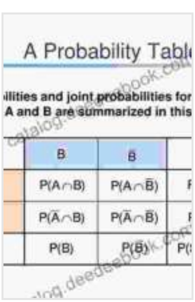
Fourier Transforms of Distributions and Their Inverses

The Fourier transform is a fundamental mathematical tool that plays a crucial role in various scientific and engineering disciplines. It allows us to convert a function from one domain to another, revealing essential information about its frequency components. However, the classical Fourier transform, defined for functions that satisfy определённые conditions, such as continuity and boundedness, has limitations. To overcome these limitations, mathematicians introduced the concept of distributions, which provide a more generalized framework for the Fourier transform.

Distributions

Distributions, also known as generalized functions, are objects that extend the notion of functions. They allow us to define and manipulate functions that may not be traditionally defined, such as functions with singularities or discontinuous derivatives. The space of distributions, denoted as D'(Ω),where Ω is an open set in ℝ^n, is a vector space that contains all possible distributions.



Fourier Transforms of Distributions and Their Inverses: A Collection of Tables (Probability and mathematical statistics) by Fritz Oberhettinger

★★★★★ 5 out of 5

Language	: English
File size	: 8893 KB
Screen Reader	: Supported
Print length	: 167 pages

One of the fundamental properties of distributions is that they can be integrated against test functions. Test functions, denoted as $\phi \in D(\Omega)$,are infinitely differentiable functions with compact support. This property enables us to define the Fourier transform of a distribution as follows:

$$(Fu)(\xi) = \}\geq,$$

where $u \in D'(\Omega)$ is a distribution, x and ξ are vectors in $\mathbb{R}^n$, <.> denotes the duality pairing between distributions and test functions, and $e^{-i}$is a test function.

The Fourier Inversion Theorem

A crucial result in distribution theory is the Fourier inversion theorem, which establishes a bijective correspondence between tempered distributions and functions in the Schwartz space. The Schwartz space, denoted as $S(\mathbb{R}^n)$,is a vector space of infinitely differentiable functions that decrease rapidly at infinity along with all their derivatives.

The Fourier inversion theorem states that if $u \in S'(\mathbb{R}^n)$ is a tempered distribution, then its Fourier transform Fu is a function in $S(\mathbb{R}^n)$,and the inverse Fourier transform of Fu is u. In other words:

$$F^{-1}(Fu) = u.$$

Applications of Fourier Transforms of Distributions

Fourier transforms of distributions have numerous applications in various fields, including:

- **Signal processing:** Analyzing and processing signals with discontinuities or singularities, such as pulses and step functions.
- **Partial differential equations:** Solving partial differential equations using Fourier analysis, including elliptic, parabolic, and hyperbolic equations.
- **Quantum mechanics:** Describing quantum states as distributions and using Fourier transforms to compute probabilities and other observables.
- **Image processing:** Filtering and enhancing images using Fourier transforms of distributions.
- **Mathematical physics:** Studying asymptotic behavior and singularities of functions, such as in the theory of distributions and the theory of generalized functions.

Fourier transforms of distributions extend the concept of the classical Fourier transform to a more general setting, enabling us to analyze and manipulate functions with singularities and other irregularities. The Fourier inversion theorem establishes a fundamental relationship between tempered distributions and functions in the Schwartz space, providing a powerful tool for solving problems in various scientific and engineering disciplines.

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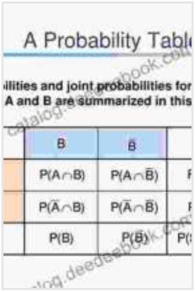

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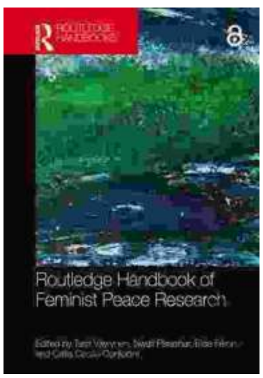

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